MBF3C Unit 4 - (Quadratics 2) - Outline

Day	Lesson Title	Specific Expectations
1	Binomial Multiplication	A1.5
2	Standard Form of a Quadratic Relation	A1.6, A1.8
3	Changing To and From Standard Form	A1.6
4	Factoring Quadratic Expressions	A1.7
5	Factoring and Solving Quadratic Expressions	A1.7, A1.9
6	Problem Solving with Quadratics	A1.9
7	Review Day	
8	Test Day	
TOTAL DAYS:		8

A1.5 – expand and simplify quadratic expressions in one variable involving multiplying binomials [e.g., 1/2 x + 1(3x – 2)] or squaring a binomial [e.g., $5(3x – 1)^2$], using a variety of tools (e.g., paper and pencil, algebra tiles, computer algebra systems);

A1.6 – express the equation of a quadratic relation in the standard form $y = ax^2 + bx + c$, given the vertex form $y = a(x – h)^2 + k$, and verify, using graphing technology, that these forms are equivalent representations [Sample problem: Given the vertex form $y = 3(x – 1)^2 + 4$, express the equation in standard form. Use technology to compare the graphs of these two forms of the equation.];

A1.7 – factor trinomials of the form $ax^2 + bx + c$, where a = 1 or where a is the common factor, by various methods;

A1.8 – determine, through investigation, and describe the connection between the factors of a quadratic expression and the x-intercepts of the graph of the corresponding quadratic relation (Sample problem: Investigate the relationship between the factored form of $3x^2 + 15x + 12$ and the x-intercepts of $y = 3x^2 + 15x + 12$.);

A1.9 – solve problems, using an appropriate strategy (i.e., factoring, graphing), given equations of quadratic relations, including those that arise from real-world applications (e.g., break-even point) (Sample problem: On planet X, the height, h metres, of an object fired upward from the ground at 48 m/s is described by the equation $h = 48t – 16t^2$, where t seconds is the time since the object was fired upward. Determine the maximum height of the object, the times at which the object is 32 m above the ground, and the time at which the object hits the ground.).

Unit 4 Day 1: Binomial Multiplication		MBF 3C
	Description Students will learn how to multiply two binomials	Materials Algebra tiles (if desired) BLM 4.1.1
		Assessment Opportunities
Minds On…	**Whole Class → Discussion** Model the distributive law… in the guise of a handshake on the corner (i.e. Jane and Reggie meet Yukio and Hatsumo and they shake hands…) This distributive law makes sense because a couple wouldn't shake hands with themselves, but they would each shake hands with each of the other couple members. Also discuss what "squaring something" means. (i.e. itself multiplied by itself!)	
Action!	**Teacher→ Directed Instruction** On board or overhead demonstrate various examples. (i.e. diagram, FOIL or algebra tiles or …) If Algebra tiles are to be used, each student needs a set and model how to use with overhead or magnetic tiles. Example 1: $(x + 3)(x + 2) = x^2 + 3x + 2x + 6$ $= x^2 + 5x + 6$ Example 2: $(x - 5)(x + 3) = x^2 - 5x + 3x - 15$ $= x^2 - 2x - 15$ Example 3: $(x-4)\left(x-\frac{1}{2}\right) = x^2 - 4x - \frac{1}{2}x + 2$ $= x^2 - \frac{9}{2}x + 2$ Example 4: $(2x + 1)(x + 2) = 2x^2 + x + 4x + 2$ $= 2x^2 + 5x + 2$ Example 5: $(x + 4)^2 = (x + 4)(x + 4)$ $= x^2 + 4x + 4x + 16$ $= x^2 + 8x + 16$ Example 6: $2(x + 1)^2 = 2(x + 1)(x + 1)$ $= 2(x^2 + x + x + 1)$ $= 2(x^2 + 2x + 1)$ $= 2x^2 + 4x + 2$	
Consolidate Debrief	**Pairs → Skill Practice** In pairs, each student makes up their own binomial question and answers it. They then tell their partner the question, receive a question from their partner, and each partner answers the new questions. They then check answers. If they don't agree, they should check their answers first to find the problem, or else involve the teacher for help.	
Concept Practice	**Home Activity or Further Classroom Consolidation** Students complete BLM 4.1.1	

MBF 3C
BLM 4.1.1

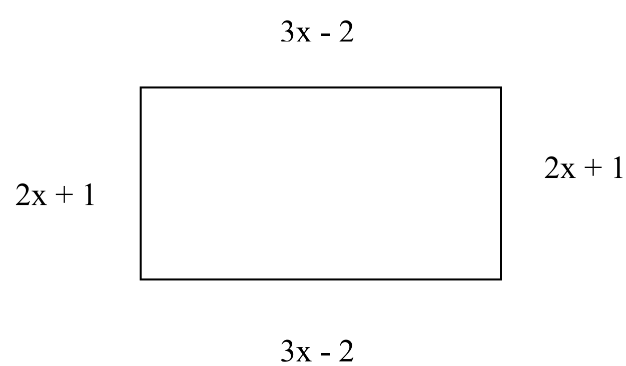
Name :
Date :

Binomial Multiplication

A. Expand the following.

1. $(x + 3)(x + 4)$	2. $(x - 3)(x + 4)$	3. $(x - 2)(x - 3)$
4. $(x+4)\left(x+\frac{1}{4}\right)$	5. $(x - 3)^2$	6. $(2x + 3)(3x - 1)$
7. $4(x - 2)^2$	8. $5(3x - 1)^2$	9. $\left(\frac{1}{2}x+1\right)(3x - 2)$

B. Using the formulas for area and perimeter of a rectangle, find the area and perimeter of the rectangle shown.



C. Using your skills of binomial multiplication expand the following.

1. $y = (x - 3)(x + 5)$

2. $y = 2(x - 3)^2 + 5$

3. $y = -3(x + 1)^2 + \frac{1}{2}$

Unit 4 Day 2:Standard Form of a Parabola			MBF 3C
	Description Students will identify standard form of a parabola Students will recognize standard form of a parabola as an equivalent way of expressing a quadratic relation. Students will learn that a parabola may be able to be expressed as three different equations		Materials BLM 4.1.1 to BLM 4.1.3
			Assessment Opportunities
Minds On…	**Pairs → Brainstorm** Have students use the graphing calculator to enter the equation $y = 2(x+3)(x+5)$ using the solid curve and $y = 2(x+4)^2 - 2$ using the solid curve with tracer. **Whole Class → Discussion** Discuss the two parabolas and if students have not come up with the answer [They are the same parabola] the teacher leads them to the revelation by asking, what the zeroes of each are, what the graph looks like, what the vertex is, etc.		To change the solid curve with the tracer on the calculator, Press Y= key cursor to the left of the y= and press enter until you see —O .
Action!	**Individuals → Explore** Students receive BLM 4.2.1 and learn about standard form. **Whole Class → Discussion** Display BLM4.2.1 on overhead and review the main ideas: Standard form will give you (1) direction of opening (2) step pattern (3) y – intercept. Ask: What information does standard form give you about a parabola? a- direction of opening c – y-intercept OPTIONAL: As a lead in to a future day, have students go back and write beside the graph the equation of the parabola in vertex and factored form!		
Consolidate Debrief	**Individuals → Practice** Students work on BLM 4.2.2 to refresh polynomial multiplication (i.e. a monomial against a binomial, trinomial, etc) This can also be used as a refresher or introduction on how to use algebra tiles if the teacher will be using them for binomial multiplication and factoring. The teacher prefaces this sheet with an explanation that these skills will help use change some forms of parabolas into others.		
Concept Practice Skill Drill	**Home Activity or Further Classroom Consolidation** Students complete BLM 4.1.3 to consolidate their skills.		

MBF3C
BLM 4.2.1

Name :
Date :

Exploring the Standard Form of a Parabola

In this investigation you will graph different parabolas and determine the information about the equation of a quadratic relation in “standard form”.

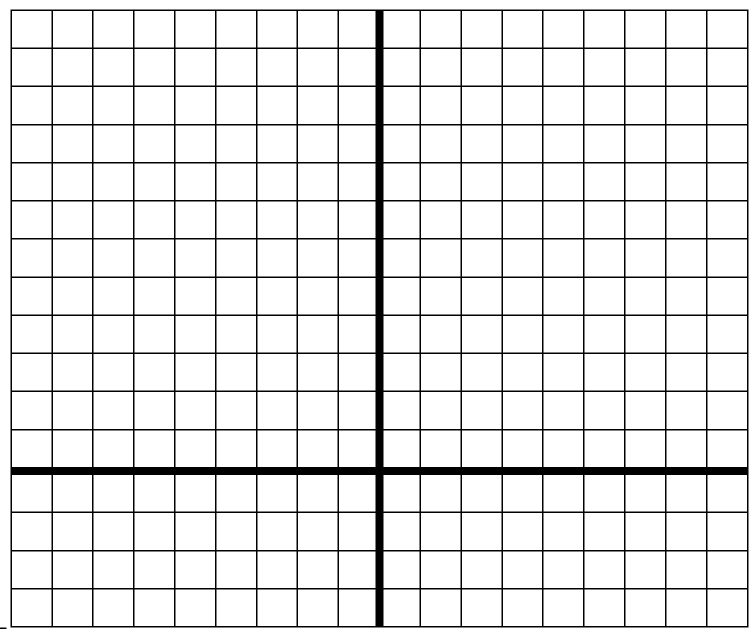
You will need to be able to determine the following about a parabola:
The y - intercept
The direction of opening
The step pattern

TECHNOLOGY OPTION
To help you graph and plot the parabolas, enter the equation in the $\boxed{Y =}$ screen on your TI – 83 graphing calculator, press $\boxed{\text{graph}}$ to see the graph and press $\boxed{\text{2nd}}$ $\boxed{\text{graph}}$ to see a table of values for the parabola

Parabola Investigation #1

Equation	$y = x^2 - 2x - 3$
Table of Values	
x	**y**
-2	
-1	
0	
1	
2	
3	
4	

Fill in the following information about the parabola:



What is the Direction of Opening?	What is the step pattern?	What is the y-intercept?
______________	_____, _____, _____	______________

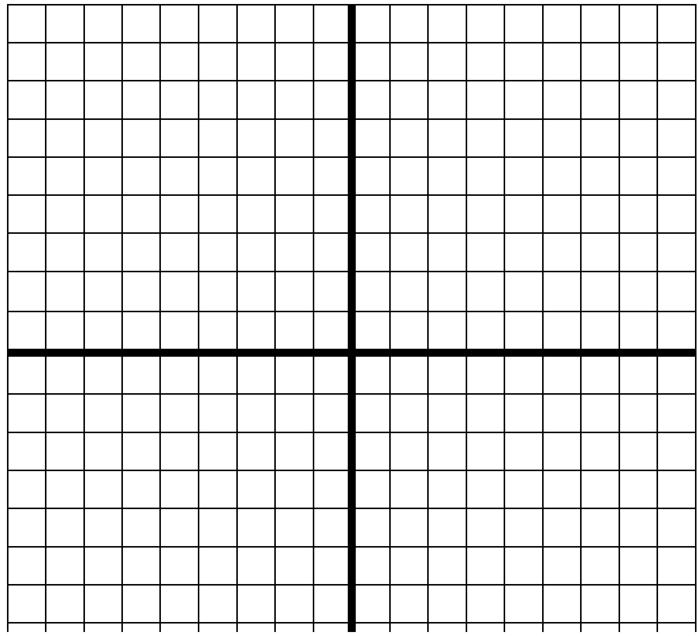
What do you notice about the y-intercept and the equation?

MBF3C
BLM4.2.1

Parabola Investigation #2

Name:
Date:

Equation	**$y = -2x^2 + 12x - 10$**
Table of Values	
x	**y**
0	
1	
2	
3	
4	
5	
6	
Fill in the following information about the parabola:	

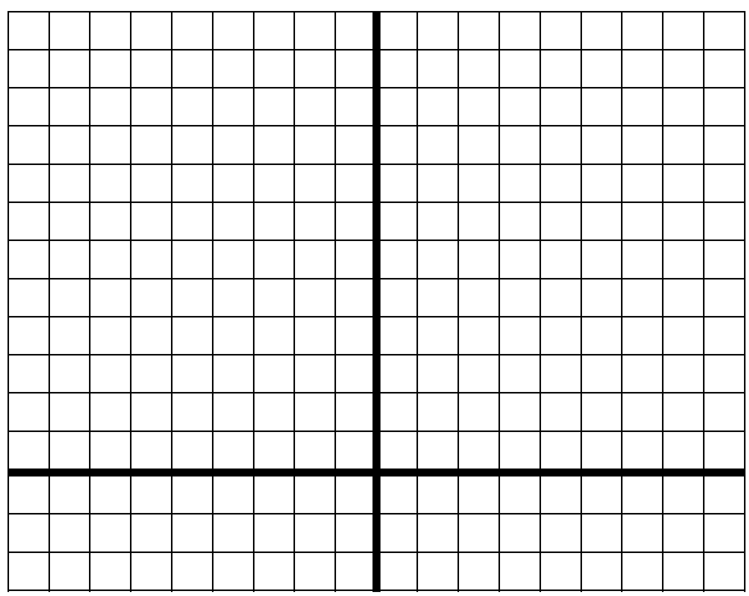


What is the Direction of Opening?	What is the step pattern?	What is the y-intercept?
______________	_____, _____, _____	______________

How does the y – intercept relate to the equation?

Parabola Investigation #3

Equation	**$y = 3x^2 + 12x + 9$**
Table of Values	
x	**y**
-5	
-4	
-3	
-2	
-1	
0	
1	
Fill in the following information about the parabola:	



What is the Direction of Opening?	What is the step pattern?	What is the y-intercept?
______________	_____, _____, _____	______________

MBF 3C
BLM 4.2.2

Name :
Date :

Standard Form of a Parabola

Standard Form of a Quadratic Relation:

$$y = ax^2 + bx + c$$

This controls the direction and opening as well as the step pattern (same as in vertex form and factored form!)

This number is the y – intercept! In this case, the y – intercept would be (0, c)

Getting to Standard Form…

In order to get to standard form, some algebra is required. Let's review what we know about polynomial multiplication:

Examples: Expand.

$(x + 3)^2$ $3(x – 4)^2$ $3(x – 4)(x + 2)$

Practice: Expand the following expressions.

1. $(x - 5)(x + 2)$	2. $2(x – 1)^2 + 3$	3. $2(x – 3)(x + 1)$
4. $2(x + 4)^2 - 3$	5. $3(x + 2)(x – 1)$	6. $– (x – 3)^2 - 2$

MBF 3C
BLM 4.2.3

Name :
Date :

Quadratics 2 Practice

1. Match each expression in the left column with one in the right column. (Hint: simplify)

1. $2x(x - 3)$	A. $2x + 8$
2. $4x - 2(x - 4)$	B. $x^2 + 3 - (3 + 6x - x^2)$
3. $3(x^2 - 4x + 2)$	C. $x(x - 6) + 2(x^2 - 3x + 1)$

2. a) Expand to express $y = 2(x - 3)^2 - 2$ in standard form.

   b) Expand each of the following and compare with the equation found in a).

   $y = -(x - 2)(x - 4)$ $\qquad$ $y = 3(x - 4)(x + 2)$

   $y = 2(x - 4)(x - 2)$ $\qquad$ $y = 2(x - 3)(x + 1)$

   c) By comparing the expanded form of the equations in 2a and 2b find the two quadratics that represent the same parabola.

3. List all the information you can about the parabola $y = 2x^2 - 4x - 6$ and then find its match (in another form, of course) in question #2

Unit 4 Day 3: Changing to and from Standard Form		MBF 3C
	Description Students will learn the equivalence of factored, vertex and standard forms. Students will gain appreciation for the reasons for factoring trinomials.	Materials Graphing calculators, BLM 4.3.1 to 4.3.3
		Assessment Opportunities
Minds On…	**Whole Group → Discussion** As students enter the classroom direct them to the two equations on the board under which is the question: Are they the same? The equations: $y = 2(x - 2)(x - 4)$ and $y = 2(x - 3)^2 - 2$ The discussion should develop that to prove they are equivalent there are two basic choices: graph them and compare, or expand them and compare.	
Action!	**Individuals → Explore** Students work through BLM 4.3.1 and BLM4.3.2 to solidify their understanding of the equivalence of the forms.	
Consolidate Debrief	**Whole Class → Discussion** Emphasize key aspects of BLM 4.3.1 and BLM 4.3.2 with the class.	
Concept Practice	**Home Activity or Further Classroom Consolidation** Students complete BLM 4.3.3 for consolidation	

MBF 3C
BLM 4.3.1

Name :
Date :

Same Parabola, Different Equation

For each of the following parabolas,

1. Expand the equation to standard form.
2. Using a graphing calculator graph each equation. (Enter the equations in the o screen, with the first equation as Y1 and the second (expanded) equation as Y2) (Remember to use the tracing curve for the second graph —○)
3. If you have expanded the equation properly you should see the second parabola being graphed on top of the first parabola.
4. If you see two parabolas… go back and check your algebra!

	Original Equation	Standard Form (show your work)	Same parabola? (yes/no)
A	$y = 2(x - 3)(x + 4)$		
B	$y = -3(x + 1)(x + 2)$		
C	$y = -(x - 6)^2 + 12$		
D	$y = (x + 5)^2 - 1$		

Therefore, when vertex or factored form is expanded to standard form it still represents the SAME PARABOLA!

MBF 3C
BLM 4.3.2

Name :
Date :

Why Standard Form?

The information about a parabola given by standard form is not as useful as the information given by vertex or factored form. So why do we use standard form?

1. Finding y-Intercepts

You can always find a y-intercept by substituting in a value of 0 for x, but if you forget this you can use standard form as well.

Example: Find the y-intercepts of $y = 2(x - 3)^2 - 14$

Solution:

Method 1: Substitute zero for x	Method 2: Expand and use equation
$y = 2(x - 3)^2 - 14$ $y = 2(0 - 3)^2 - 14$ *Sub $x = 0$* $y = 2(-3)^2 - 14$ $y = 2(9) - 14$ $y = 18 - 14$ $y = 4$ **The y – intercept is (0, 4)**	$y = 2(x - 3)^2 - 14$ $y = 2(x - 3)(x - 3) - 14$ $y = 2(x^2 - 3x - 3x + 9) - 14$ $y = 2(x^2 - 6x + 9) - 14$ $y = 2x^2 - 12x + 18 - 14$ $y = 2x^2 - 12x + 4$ **The y – intercept is (0, 4)**

Practice:

Use both methods above to find the y-intercept of $y = (x + 2)^2 - 9$

2. Finding Points

If you know the x-value of a point on the parabola you can use the parabola's equation to find the y-value simply by substituting in the value for x and evaluating. BUT… if you know the y-value and want the x-value the algebra takes more work and standard form is simpler to use in this case. ** We will see more of this in a later lesson!**

3. Easier Algebra

Standard form is a "simplified" parabola equation since there are no brackets, or exponents, to be expanded. This makes certain algebraic tasks easier to accomplish. For example, a computer or graphing calculator can calculate points on a parabola more efficiently using standard form than using vertex or factored form.

MBF 3C
BLM 4.3.3

Name :
Date :

Standard Form of a Quadratic Relation

1. Write the following quadratic relations in standard form.

(a) $y = (x + 2)(x + 1)$	(b) $y = (x - 3)^2 - 3$	(c) $y = 2(x - 3)(x + 4)$
(d) $y = 2(x - 3)^2 - 1$	(e) $y = -3(x - 2)(x + 3)$	(f) $y = -(x + 2)^2 - 7$

2. “When a parabola has its vertex on the y – axis the equation looks the same in vertex form and in standard form.” Is this true? Provide an example as proof.

Unit 4 Day 4: Factoring Quadratic Expressions		MBF 3C
	Description Students will learn to factor quadratic expressions of the form $x^2 + bx + c$	Materials Algebra tiles if needed, BLM 4.4.1
		Assessment Opportunities

Minds On…

Pairs → Share

Students answer a list of questions from the board:

$(x-2)(x+3) = x^2 + x +$ ____ [-6]
$(x+4)(x+3) = x^2 + 7x +$ ____ [12]
$(x+7)(x+3) = x^2 +$ ____ $x +$ ____ [10, 21]
$(x-5)(x+1) = x^2 +$ _____ $x +$ ____ [-4, -5]

$(x +$ ____$)(x+2) = x^2 + 3x + 2$ [1]
$(x+2)(x +$ ___$) = x^2 - x - 6$ [-3]
$(x +$ ____$)(x +$ ____$) = x^2 + 6x + 8$ [2, 4]

Each pair when done teams up with another group to compare answers and then they add another pair until hopefully the entire class has the same answers.

The teacher then takes up the answer and explains to the students that the final question is the object of the day's lesson and it involves REVERSING binomial multiplication

This material has been taught in grade 10 but will probably need a lot of review.

Action!

Whole Class → Discussion

Algebra tiles may be used here to demonstrate factoring, or perhaps a summary of the product/sum method.

Examples:

FACTOR

a) $x^2 + 5x + 6$ b) $x^2 + 3x - 18$ c) $x^2 - 9x + 20$ d) $x^2 - 10x + 16$

e) $x^2 + 8x + 16$ f) $x^2 - 17x + 72$ g) $x^2 + 2x - 48$

Answers: a) $(x+2)(x+3)$ b) $(x+6)(x-3)$ c) $(x-4)(x-5)$ d) $(x-2)(x-8)$ e) $(x+4)(x+4)$ f) $(x-8)(x-9)$ g) $(x+8)(x-6)$

Consolidate Debrief

Pairs → Skills Practice

Each member of a pair makes their own question (for example $(x + 2)(x - 4)$ and then expands it. ($x^2 - 2x - 8$). The expanded expressions are exchanged and the students factor the new equation and check the original expression for the correct answer. This can become a game with the students trying to outdo each other (it might help to keep the numbers as integers and between 10 and -10)

Concept Practice
/

Home Activity or Further Classroom Consolidation

Students complete BLM 4.4.1

MBF 3C | Name :
BLM 4.4.1 | Date :

Factoring Quadratic Expressions

1. Fill in the missing numbers.

   (a) $(x - 3)(x + 4) = x^2 + x +$ ____

   (b) $(x - 6)(x + 2) = x^2 +$ ______ $x +$ ____

   (c) $(x +$ ____ $)(x + 2) = x^2 + 5x + 6$

   (d) $(x + 3)(x +$ ___ $) = x^2 - 6x - 27$

   (e) $(x +$ ____ $)(x +$ ____ $) = x^2 + 9x + 14$

2. Factor each expression.

(a) $x^2 - 3x - 4$	(b) $x^2 - 11x + 28$	(c) $x^2 + 7x + 12$
(d) $x^2 - 4x - 32$	(e) $x^2 - 13x + 42$	(f) $x^2 - 4x + 4$

3. Connecting to prior lessons, by factoring standard form, we can change a parabola's equation into factored form!

   Given the equation: $y = x^2 + 8x + 15$

(a) state the y – intercept ______________

(b) write the expression in factored form $y =$_______________

(c) the zeros of the parabola are __________ and ____________

(d) the vertex of the parabola is _________________
(hint: the vertex is located halfway between the zeros)

(e) the axis of symmetry of the parabola is _______________

Unit 4 Day 5: Factoring and Solving Quadratic Expressions			MBF 3C
	Description Students will learn to factor quadratic expressions of the form $ax^2 + bx + c$ where a is a common factor Students will learn to use factoring as a strategy in problem solving.		Materials BLM 4.5.1 and BLM 4.5.2
			Assessment Opportunities
Minds On…	**Whole Class → Discussion** A quadratic expression is on an overhead or the board: $2x^2 - 4x - 16$ How is this different from yesterday's work? [there is a number in front of x^2] How can this be dealt with? [common factor it out] If the answer to the second question is difficult to draw out, start with factored form and highlight the common factor point. $y = 2(x - 4)(x + 2)$ $y = 2(x^2 - 4x + 2x - 8)$ $y = 2(x^2 - 2x - 8)$ $y = 2x^2 - 4x - 16$ Highlight where common factoring occurs and where trinomial factoring occurs. Next challenge: Put $y = 2x^2 - 6x$ in factored form		
Action!	**Pairs → Skill Practice** Similar to yesterday's work, the students prepare their own factored form with an "a" value (i.e. $y = 3(x - 3)(x + 5)$ and then expand it and give that as a question to their partner. **Whole Class → Discussion** Ask for varying levels of difficulty from the class (i.e. "Let's try one as a class that you thought was pretty easy, or pretty tough," etc) Model 4 or 5 questions on the board to help consolidate the ideas. **Individuals → Application Practice** Students work on BLM 4.5.1.		
Consolidate Debrief	**Whole Class → Discussion** Take up BLM 3.5.1.		
Application	**Home Activity or Further Classroom Consolidation** Students complete BLM 4.5.2		

MBF 3C
BLM 4.5.1

Name :
Date :

Finding Factored Form!

We can use trinomial factoring to change standard form to factored form to answer many different types of problems.

1. A parabola has the equation $y = 2x^2 - 4x - 6$

(a) write the equation in factored form

____

(b) determine the zeroes

____

(c) determine the axis of symmetry

____

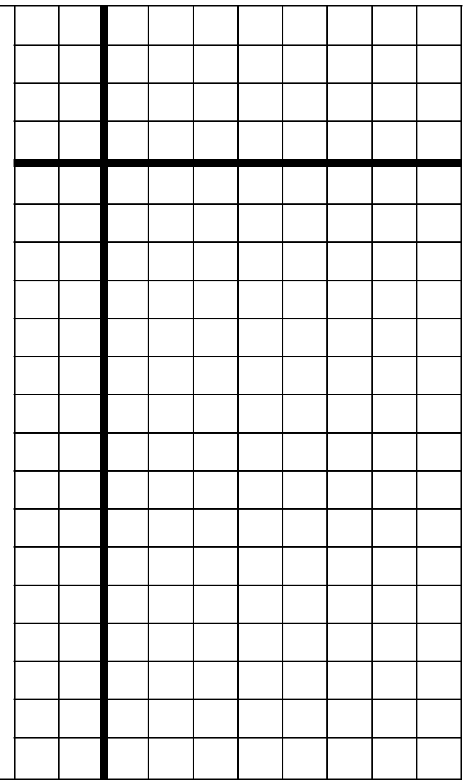
(d) determine the vertex _________________

(e) determine the step pattern

____

(f) graph the parabola at the right

(g) write the equation of the parabola in the vertex form



2. Imagine you had a quadratic relation in standard form. What steps would you take to make an accurate graph of the parabola?

3. A ball is thrown upwards. Its height is described by the equation $h = -5t^2 + 20t$, where h is measured in meters and t is measured in seconds.

(a) Common factor $h = -5t^2 + 20t$,

(b) how high is the ball at 0, 1, and 2 seconds?

(c) Using the factored expression find the time when the ball hits the ground (hint: when h=0)

(d) use your answers from (c) to find the maximum height of the ball and when it occurs.

MBF 3C
BLM 4.5.2

Name :
Date :

Quadratics Assignment

1. A parabola has the equation $y = -2x^2 + 12x - 10$

(a) write the equation in factored form

____

(b) determine the zeroes

____

(c) determine the axis of symmetry

____

(d) determine the vertex

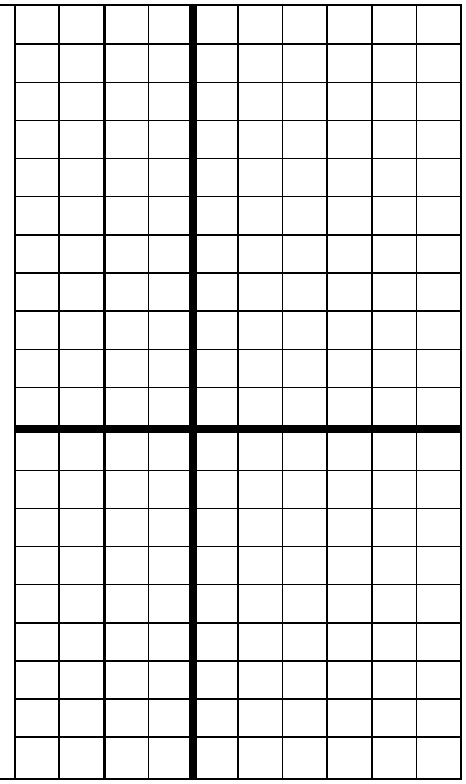
____

(e) determine the step pattern

____

(f) graph the parabola at the right

(g) write the equation of the above parabola in vertex form



2. A cannonball is launched upwards. Its height is described by the equation $h = -5t^2 + 40t + 45$, where h is measured in yards and t is measured in seconds.

a) how high is the cannonball at 0, 1, and 2 seconds?

b) from what height was the cannonball launched?

c) factor the expression to find when the cannonball hits the ground

d) use your answers from (c) to find the maximum height of the cannonball and when it occurs.

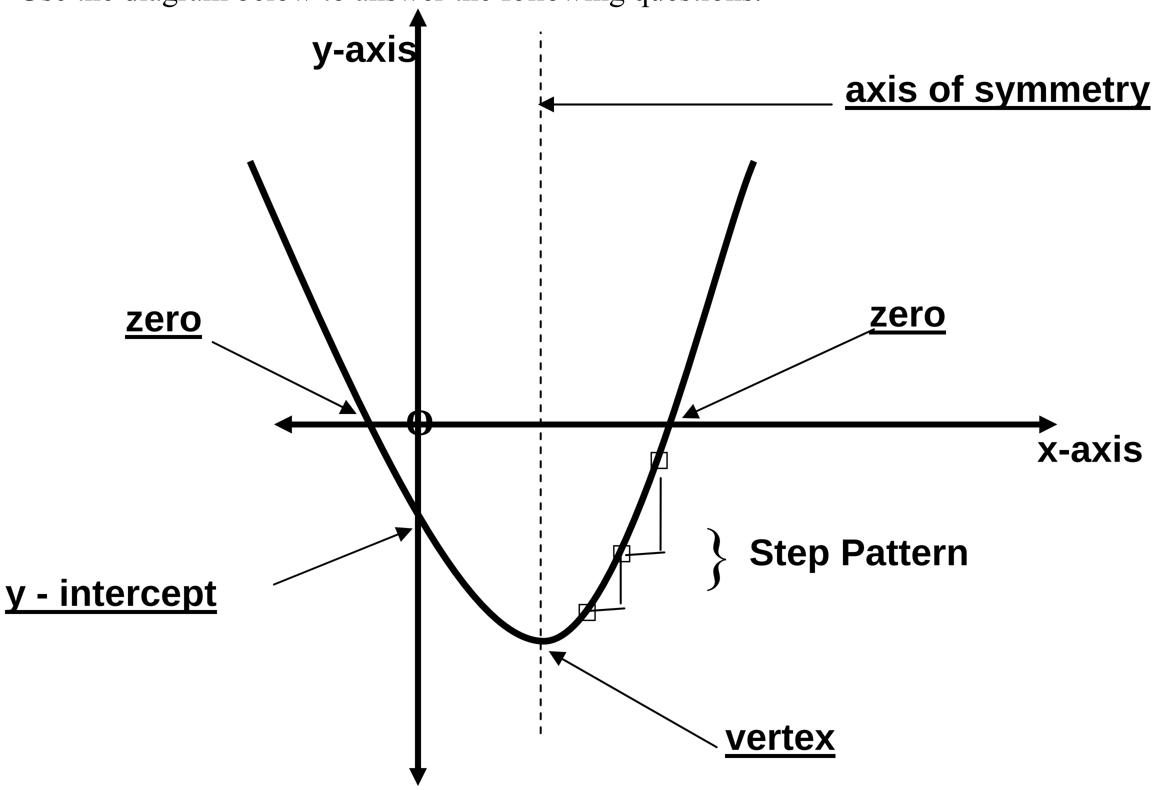
Unit 4 Day 6: Problem Solving With Quadratics		MBF 3C
	Description Students will solve problems using appropriate strategies.	Materials BLM 4.6.1 to BLM 4.6.4 Scissors, glue, blank paper
		Assessment Opportunities
Minds On…	**Pairs/Whole Class → Brainstorm** In pairs work through the summarization of what they have learned thus far about quadratics on BLM 4.6.1. The students then take up the answers to create a master list on the board. Note that a lot of work was repeated in the solutions. Pose the question: What would be the best order to do parts (a) through (d) from BLM4.6.1?	
Action!	**Groups → Application** Two pairs form a group. Post the problem (BLM 4.6.1) on the overhead and each group gets a slip of paper identifying one of the parts (a) to (d) that their group will answer. Each group (starting at (a) and going to (d)) present their solution to the class.	
Consolidate Debrief	**Whole Class → Discussion** Clarify any misunderstandings from the presentations if needed. **Pairs → Practice** Students work on the problem outlined on BLM 4.6.3.	
Application	**Home Activity or Further Classroom Consolidation** Students complete BLM 4.6.4.	

MBF3C
BLM4.6.1

Name:
Date:

Summarizing Parabolas

Use the diagram below to answer the following questions.



What are the important parts of the parabola?

What information does each of the following algebraic representations of the parabola give us?

i) Factored Form ii) Standard Form iii) Vertex Form

What information do you need to graph a parabola?

How do you factor a quadratic expression?

How do you multiply two binomials?

Name:
Date:

On Planet X, the height, *h* metres, of an object fired upward from the ground at 48m/s is described by the equation $h = 48t - 16t^2$, where *t* seconds is the time since the object was fired upward.

Determine

(a) the maximum height of the object

(b) the times at which the object is 32m above the ground

(c) the time at which the object hits the ground

(d) the equation in vertex form

Cut Outs for Planet X Problem

a) Determine the maximum height of the object.

b) Determine the times at which the object is 32m above the ground.

c) Determine the time at which the object hits the ground.

d) Determine the equation in vertex form.

MBF3C
BLM 4.6.2

Solutions to Planet X Problem

Part (a)

"Maximum height" indicates that we should solve for the vertex.

$h = 48t - 16t^2$ is in standard form

Rewrite as $h = -16t^2 + 48t$

Common factor $h = -16t(t - 3)$

From factored form $h = -16t(t - 3)$ the zeroes are 0 and 3

Axis of symmetry is $t = 1.5$ (at 1.5s the object is at its max height)

Sub $t = 1.5$ into equation

$$h = -16(1.5)^2 + 48(1.5)$$

$$h = -16(2.25) + 48(1.5)$$

$$h = -36 + 72$$

$$h = 36$$

The maximum height reached by the object is 36m

Solutions to Planet X Problem

Part (b)

Solve for t when $h = 32m$

Original equation $h = 48t - 16t^2$

Sub 32 for h $32 = 48t - 16t^2$

Rearrange equation $0 = -16t^2 + 48t - 32$

Common factor $0 = -16(t^2 - 3t + 2)$

Trinomial factor $0 = -16(t - 2)(t - 1)$

The solutions are $t = 1$ or $t = 2$. Thus the object is at 32m of height at 1s and 2s.

Part (c)

The ground has a height of zero. Thus, find t when $h = 0$. Or, find the zeroes of the parabola.

$h = 48t - 16t^2$ is in standard form

Rewrite as $h = -16t^2 + 48t$

Common factor $h = -16t(t - 3)$

From factored form $h = -16t(t - 3)$ the zeroes are 0 and 3

Thus the object hits the ground at 0s (launch) and 3s.

MBF3C
BLM 4.6.2

Solutions to Planet X Problem

Part (d)

Vertex form needs vertex, step pattern and direction of opening.

$h = 48t - 16t^2$ is in standard form

Rewrite as $h = -16t^2 + 48t$

Here we see $a = -16$ (this tells us the step pattern is 16, 48, 80 and that the parabola opens down!)

Common factor $h = -16t(t - 3)$

From factored form $h = -16t(t - 3)$ the zeroes are 0 and 3

Axis of symmetry is $t = 1.5$ (at 1.5s the object is at its max height)

Sub $t = 1.5$ into equation

$$h = -16(1.5)^2 + 48(1.5)$$

$$h = -16(2.25) + 48(1.5)$$

$$h = -36 + 72$$

$$h = 36$$

The vertex is (1.5, 36) and thus vertex form is

$$h = -16(t - 1.5)^2 + 32$$

MBF 3C | Name :
BLM 4.6.3 | Date :

Quadratic Quandry!

The “Quadratics Cup” is a new coffee shop that sells various coffee-related items, but with a quadratics twist (like an equation written on your cup, or your caramel drizzle in the shape of parabolas, etc)

The profit of this new coffee shop can be described by the equation

$$P = -4n^2 + 64n - 112$$

Where P represents profit in thousands of dollars and n represents the number of customers, in ten thousands.

(a) What is the profit if no customers visit the store?

(b) What are the break-even points of the store?

(c) What is the maximum possible profit of the store?

(d) If a company makes a $80 000 profit they receive a municipal Silver Business Award. How many customers are needed in order for the Quadratics Cup to receive the award?

COMPETITION: Across the road another coffee shop has opened (It’s called the “Mathees Muffins”) It’s profit equation is

$$P = -5t^2 + 60t - 100$$

Answer (a) to (c) from above about the new coffee shop.

MBF 3C
BLM 4.6.3

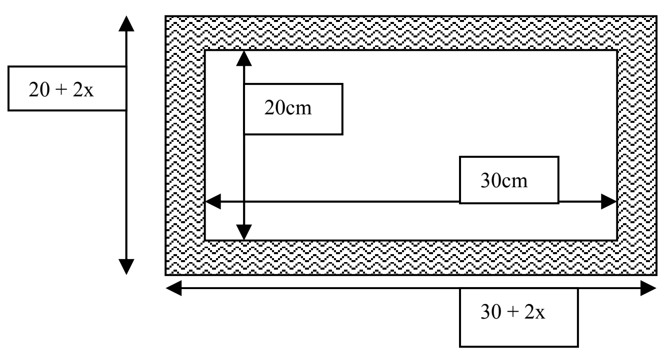
Name :
Date :

Problem Solving With Quadratics

1. A parabola has the equation $y = 2x^2 + 4x - 6$. Determine:
   (a) the zeros of the parabola
   (b) the vertex of the parabola
   (c) the points where the parabola reaches a "height" of $y = 10$
   (d) the equation of the parabola in vertex form and factored form
   (e) sketch the parabola on a piece of graph paper

2. The effectiveness of an internet pop-up is based on how many viewers it receives per day. There is an optimal number of hits and beyond this the effectiveness drops off. The effectiveness can be described by the equation $E = -2n^2 + 18n + 44$ where E represents the percent effectiveness and n represents the number of viewers in millions of people. Determine:
   (a) the maximum effectiveness and how many viewers are needed to achieve this
   (b) the number of viewers needed to make the pop-up 80% effective
   (c) when the pop-up is 0% effective

3. EXTENSION: Yasmeen is trying to calculate the width of a matte to go around a 20cm by 30cm photograph she wants to display. She has decided that the matte will have the same area as the photo (the Home Depot book suggested this). She just isn't sure how thick the matte should be. Using the diagram shown determine:
   (a) an expression for the area of the photo's matte (remember, the area would be the big rectangle subtract the small rectangle)
   (b) the area of the photograph
   (c) an equation for the area of the matte
   (d) simplify the equation in (c) (put it in "standard" form)
   (e) solve the equation in (d) and decide how thick the matte should be



MBF3C Unit 4 (Quad 2) Solutions

Day 1

BLM 4.1.1

PART A

1. $x^2 + 7x + 12$, 2. $x^2 + x - 12$, 3. $x^2 - 5x + 6$, 4. $x^2 + \frac{17}{4}x + 1$, 5. $x^2 - 6x + 9$,

6. $6x^2 + 7x - 3$, 7. $4x^2 - 16x + 16$, 8. $45x^2 - 30x + 5$, 9. $\frac{3}{2}x^2 + 2x - 2$

PART B

Area = $6x^2 - x - 2$, perimeter = $10x - 2$

PART C

1. $y = x^2 + 2x - 15$, 2. $2x^2 - 12x + 23$, 3. $y = -3x^2 - 6x - 5/2$

Day 2

BLM 4.2.1

Parabola Investigation #1:
y-values: 5, 0, -3, -4, -3, 0, 5
Direction of opening: up
Step Pattern: 1, 3, 5, …
Y-intercept: -3

Parabola Investigation #2:
y-values: -10, 0, 6, 8, 6, 0, -10
Direction of opening: down
Step Pattern: -2, -6, -10
Y-intercept: -10

Parabola Investigation #3:
y-values: 24, 9, 0, -3, 0, 9, 24
Direction of opening: up
Step Pattern: 3, 9, 15, …
Y-intercept: 9

Standard form gives you the step pattern, direction and opening and the y-intercept.

BLM 4.2.2

Examples: $x^2 + 6x + 9$, $x^2 - 24x + 48$, $3x^2 - 6x - 24$

Practice: 1. $x^2 - 3x - 10$, 2. $2x^2 - 4x + 5$, 3. $2x^2 - 4x - 6$,
4. $2x^2 + 16x + 29$, 5. $3x^2 + 3x - 6$, 6. $-x^2 + 6x - 11$

MBF3C **Unit 4 (Quad 2) Solutions(Continued)**

BLM 4.2.3

1.

1. $2x(x-3)$ $2x^2-6x$	A. $2x+8$
2. $4x-2(x-4)$ $2x+8$	B. $x^2+3-(3+6x-x^2)$ $2x^2-6x$
3. $3(x^2-4x+2)$ $3x^2-12x+6$	C. $x(x-6)+2(x^2-3x+1)$ $3x^2-12x+6$

2a) $y = 2x^2 - 12x + 16$

b) $y = -x^2 + 6x - 8$; $y = 3x^2 - 6x - 24$; $y = 2x^2 - 12x + 16$; $y = 2x^2 - 4x - 6$

The third expression simplifies to the same result as in a).

c) $y = 2(x-4)(x-2)$ has the same parabola as the one in a) the both opens up, have a step pattern 2, 6, 10, .. and y – intercept of –6…

Day 3

BLM 4.3.1

A. $y = 2x^2 + 2x - 12$, B. $y = -3x^2 - 9x - 6$, C. $y = -x^2 + 12x - 24$, D. $y = x^2 + 10x + 26$

BLM 4.3.2

1. The y-intercept is (0, -5)
3. The equation of the ultra function is $U = x^2 + 18x - 10$

BLM 4.3.3

1(a) $y = x^2 + 3x + 2$, (b) $y = x^2 - 6x + 6$, (c) $y = 2x^2 + 2x - 24$, (d) $y = 2x^2 - 12x + 8$, (e) $y = -3x^2 - 3x + 18$, (f) $y = -x^2 - 4x - 11$

2. Yes. Answers will vary

Day 4

BLM 4.4.1

1. (a) –12, (b) –4, -12, (c) 3, (d) -9, (e) 2, 7
2. (a) $(x-4)(x+1)$, (b) $(x-7)(x-4)$, (c) $(x+3)(x+4)$, (d) $(x+4)(x-8)$ (e) $(x-6)(x-7)$, (f) $(x-2)2$
3. (a) (0, 15), (b) $y = (x+3)(x+5)$, (c) -3 and –5, (d) (-4, -1) (e) $x = -4$

MBF3C **Unit 4 (Quad 2) Solutions(Continued)**

Day 5

BLM 4.5.1

1. (a) $y = 2(x – 3)(x + 1)$, (b) 3 and –1, (c) $x = 1$, (d) (1, -8), (e) 2, 6, 10…
   (g) $y = 2(x – 1)^2 -8$
2. from a-value: direction of opening, step pattern. Then factor and get zeros. From zeros get axis of symmetry and then sub in to get optimal value (vertex). Then graph!
3. (a) $h = -5t(t – 4)$ b) 0 m, 15m, 20m c) 4 seconds d) 20m

BLM 4.5.2

1. (a) $y = -2(x – 5)(x – 1)$, (b) 5 and 1, (c) $x = 3$, (d) (3, 8), (e) -2, -6, -10…
   (g) $y = -2(x – 3)2 + 8$
2. (a) 105 yards b) 45 yards c) at -1 and 9 seconds d) 125 yards at 4 seconds

Day 6

BLM4.6.3

Quadratic Quandry (a) -$112 000, (b) 20 000 and 140 000 people, (c) $144 000, (d) 40 000 and 120 000 people

Competition (a) -$100 000, (b) 20 000 and 100 000 people, (c) $80 000

BLM 4.6.4

1. (a) -3 and 1, (b) (- 1, -8), (c) (2, 10) and (-4, 10) (d) $y = 2(x + 1)^2 – 8$ and $y = 2(x + 3)(x – 1)$
2. (a) 84.5% at 4.5 million viewers, (b) 3 million and 6 million (c) -2 and 11 million
3. (a) $(20 + 2x)(30 + 2x) – (30)(20) = 4x^2 + 100x$, (b) 600, (c) $4x^2 + 100x = 600$, (d) $0 = 4x^2 + 100x – 600$, (e) 5 cm thick